WiSe 2024/25
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Algebraic and Topological Methods in Discrete Mathematics

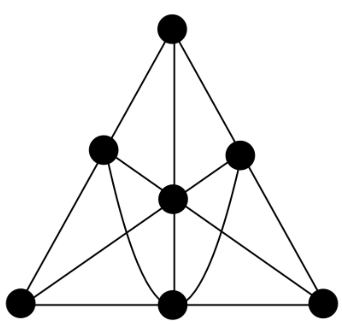
Finite reflection groups, hyperplane arrangements, and (oriented) matroids

12. Homework sheet

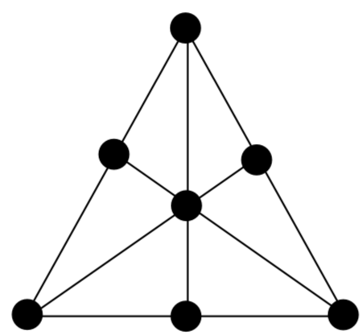
Problem 1.
(a) Find a matroid in which equality does not always hold in (R3).
(b) Find a *representable* matroid in which equality does not always hold in (R3).
(c) Find all matroids for which equality does always hold in (R3).

Problem 2. For $m \leq n$ let $U_{m,n} = (E, \mathcal{I})$ be the uniform matroid where E is the set $\{1, \ldots, n\}$ and $\mathcal{I}$ are all subsets of E of size at most m.
(a) Determine which uniform matroids are respresentable over some field.
(b) Determine which uniform matroids are transversal.
(c) Determine which uniform matroids are graphic.

Problem 3. Recall the Fano matroid and the non-Fano matroid defined via these diagrams as discussed in the lecture.



(A) The Fano matroid.



(B) The non-Fano matroid.

Prove the following two statements.
(a) The Fano matroid is representable over a field $\mathbb{F}$ if and only if $\mathrm{char}(\mathbb{F}) = 2$.
(b) The non-Fano matroid is representable over a field $\mathbb{F}$ if and only if $\mathrm{char}(\mathbb{F}) \neq 2$.

Hint: a vector configuration representing a matroid can be written in a matrix where the columns are indexed by E. Applying Gaussian elimination (using row operations) does not change the matroid this matrix represents. So we can assume that the columns corresponding to one fixed basis are the identity matrix.

Problem 4. Let G be a bipartite graph with bipartition (D, E). Let $\mathcal{I}$ be the collection of subsets of E that can be matched to D. Prove that $(E, \mathcal{I})$ is a matroid, i.e., it satisfies the axioms (I1)-(I3).