Algebraic and Topological Methods in Discrete Mathematics

Finite reflection groups, hyperplane arrangements, and (oriented) matroids

8. Homework sheet

Problem 1. Let $\mathcal{A}$ be the boolean arrangement given by $Q(\mathcal{A}) = x_1 \cdots x_\ell$. Prove that $\mu(X) = (-1)^{r(X)}$ for $X \in L(\mathcal{A})$. Compute the characteristic polynomial and the number of chambers of $\mathcal{A}$.

Problem 2. Compute $\chi_{\mathcal{A}}(q)$ for the arrangement in $\mathbb{F}_q^\ell$ defined by

$$Q(\mathcal{A}) = \prod_{1\leq i\leq j\leq \ell} (x_i + \ldots + x_j).$$

Deduce a formula for the characteristic polynomial $\chi_{\mathcal{A}}$ of the braid arrangement.

Problem 3. Let $\mathcal{A}$ be an arrangement, $H_0 \in \mathcal{A}$, and $\mathcal{A}'' := \mathcal{A}^{H_0}$ be the restriction of $\mathcal{A}$ to H_0. Show that for $\mathcal{B}'' \subseteq \mathcal{A}''$,

$$(-1)^{|\mathcal{B}''|} + \sum_{H_0\in\mathcal{B}\subseteq\mathcal{A},\, \mathcal{B}^{H_0}=\mathcal{B}''} (-1)^{|\mathcal{B}|} = 0.$$