

Algebraic and Topological Methods in Discrete Mathematics
Finite reflection groups, hyperplane arrangements,
and (oriented) matroids

1. Homework sheet

- Problem 1.** Let $\sigma \in \mathfrak{S}_n$ be a permutation.
- Show that $\ell(\sigma) = \text{inv}(\sigma)$.
 - Find a procedure to generate all reduced expressions.
- Problem 2.** Let W be a finite reflection group in \mathbb{R}^2 . Let $v \neq 0$ be contained in one of the reflecting hyperplanes. Show that the orbit Wv is a regular polygon and W its symmetry group.
- Problem 3.** Consider two infinitely long walls meeting in a corner at an angle $\alpha \in (0, \pi]$. Show that any kicked ball (which doesn't lose momentum) can meet the walls only a finite number of times. What is the maximal number of times a ball can hit the walls?
- Problem 4.** Let $S \subset \mathbb{R}^n$ a finite set such that $\text{span}(S) = \mathbb{R}^n$. Let G be the group of linear transformations $g \in \text{GL}(\mathbb{R}^n)$ such that $gS = S$. Show that G is a finite group.